Int. J. Heat Mass Transfer. Vol. 21, pp. 849–854
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0017-9310/78/0701-0849 $02.00/0

SURFACE RENEWAL MODEL OF CONDENSATION HEAT TRANSFER IN TUBES WITH IN-LINE STATIC MIXERS

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(Received 22 June 1977 and in revised form 5 December 1977)

Abstract—A surface renewal model has been developed for condensation heat transfer inside a circular tube with in-line static mixers. This model gives rise to the following dimensionless correlating equation:

$$[Nu] = C[Pr][Re_l]\left[\frac{\bar{\mu}_l}{\mu_m}\right]\left[\frac{d}{L}\right]\left[\frac{\rho_m}{\bar{\rho}_l}\right]^{1/2}\left[\frac{(1-x)H_{fg}+H_{sp}}{C_{pm}\Delta T}\right]^{1/2}.$$

The constant C was determined experimentally.

NOMENCLATURE

A, heat-transfer area;
A_i, inside surface area of the condenser tube;
A_o, outside surface area of the condenser tube;
A_l, cross-sectional area of the condensate flow;
A_r, heat-transfer area between packets and wall;
C_{pm}, mean specific heat of the vapor–condensate mixture;
C_1, proportionality constant;
C_2, proportionality constant;
C_3, proportionality constant;
C_4, proportionality constant;
d, inside diameter of condenser tube;
D, outside diameter of condenser tube;
h_i, inside condensation heat-transfer coefficient;
h_o, coolant side heat-transfer coefficient;
H_{fg}, latent heat;
H_{sp}, sensible heat;
k, thermal conductivity of condenser tube;
k_m, mean thermal conductivity of the vapor–condensate mixture;
L, length of the condenser tube;
$\dot{M}$, mass flow rate of the condensing fluid;
N, number of static mixer units;
Nu, Nusselt number of the vapor–condensate mixture, $h_i d/k_m$;
P_l, property of the condensate at saturation temperature;
P_v, property of the vapor at saturation temperature;
Pr, Prandtl number, $\mu_m C_{pm}/k_m$;
q_i, instantaneous heat flux at the surface;
q, mean heat flux;
Q, total heat-transfer rate from the test condenser;
Re_l, Reynolds number of the condensate, $\dot{M}/\bar{\mu}_l d$;
T, temperature;
T_b, bulk temperature of the vapor–condensate packet;
T_w, temperature of the heat-transfer surface;
ΔT, temperature difference, $(T_b - T_w)$;
$\bar{t}$, mean resident time;
$\bar{u}_l$, mean velocity of the condensate;
u_m, mean velocity of the vapor–condensate mixture;
$\dot{V}_l$, volumetric flow rate of the condensate.

Greek symbols
θ, contact time;
α_m, thermal diffusivity of the vapor–condensate mixture;
$\bar{\rho}_l$, mean density of the condensate;
ρ_m, mean density of the vapor–condensate mixture;
δ, thickness of the packet;
δ_{ave}, average thickness of the packet;
$\bar{\mu}_l$, mean dynamic viscosity of the condensate;
μ_m, mean dynamic viscosity of the vapor–condensate mixture;
$\bar{\tau}$, mean period of the contact time.

INTRODUCTION

THE AUGMENTATION of condensation heat transfer is very important to many engineering systems and processes involving energy, chemical, physical and biological processing [1, 2]. Experimental evidence [3] indicates that the in-line static mixers are effective in enhancing the rate of condensation heat transfer inside a circular tube. However, no attempt has been made to derive a mechanistic model or establish a design correlation for the rate of heat transfer in such a tube.

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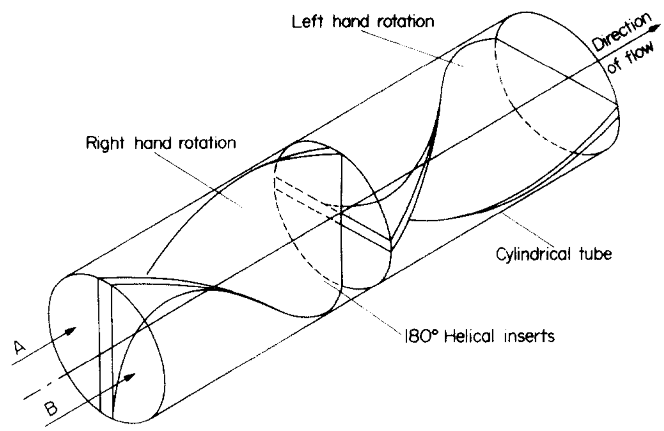

FIG. 1. Essential features of the Kenics static mixing system.

The unique characteristics of in-line static mixers [4–6] suggest that the classical boundary-layer theory is not suitable for modeling the condensation heat transfer inside a tube with in-line static mixers. An alternative approach, therefore, may be necessary. The purpose of this work is to examine the suitability of the so-called surface renewal or penetration model as an alternative approach.

FORMULATION OF THE MODEL

To develop a model for the system of the present study, an understanding of the characteristics and functions of in-line static mixers is necessary.

Static mixers were originally developed during the 1960's to mix gas–liquid, liquid–liquid, and liquid–solid in the synthetic fibers, plastics, petroleum, chemical water treatment, food, and pharmaceutical industries. One of the early static mixers was developed by the Kenics Corporation [4–6]. The mixer units are constructed of a number of short elements of right- and left-hand helices. These elements are orientated so that each leading edge is at 90° to the trailing edge of the one ahead. In general, the length of the individual element is approximately 1.5 diameters. The elements' assembly is inserted inside a tubular housing. Figure 1 shows an isometric view of the essential features of the static mixer. Because of the geometric features of static mixers, the flow stream divides at the leading edge of each element and follows the semi-circular channel created by the element's shape. At each succeeding element, the two flows are further divided, resulting in an exponential progression of flow divisions. This sequence of flow division and rotation gives rise to thorough and efficient radial mixing.

The flow pattern in a condenser tube with in-line static mixers has been observed to be homogeneous, with vapor and condensate well mixed and vigorously agitated throughout the cross-section of the tube [3]. Therefore, one can visualize the following heat-transfer mechanism [7–10].

As depicted in Fig. 2, a mass or packet containing the well-mixed vapor and condensate, originally in the main stream at the bulk temperature, T_b, is driven radially and comes into contact with the heat-transfer surface of temperature T_w. The packet is considered to have the mass average properties of the vapor and the condensate. A mean local property P_m of the packet is

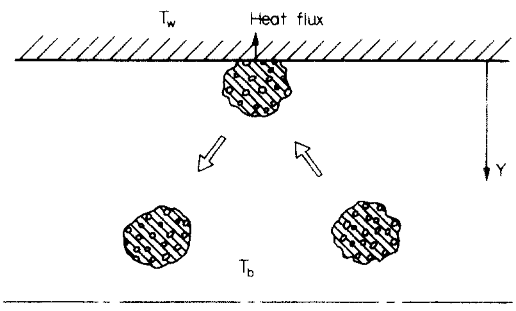

FIG. 2. Transport of a vapor–condensate packet to a heat-transfer surface.

$$P_m = \bar{x}P_v + (1-\bar{x})P_l$$

where

$\bar{x} = (1+x)/2$,

x = mass vapor quality or the mass fraction of vapor in the vapor–condensate packet at the condenser exit.

Since the condenser tube is relatively short, it can be assumed further that the local properties of the packet are essentially identical to the average properties of the vapor–condensate mixture over the entire tube.

A vapor–condensate packet remains in contact with the heat-transfer surface for a certain period of time prior to its replacement by another packet moving from the mainstream. In other words, the heat transfer is periodically "renewed". When the number of static mixers in the condenser tube is N, the total number of flow division is 2^N [4, 5]. If we assume that the mean residence time of the vapor–condensate mixture in the condenser tube is $\bar{t}$, the average frequency of the surface renewal will be proportional to $2^N/\bar{t}$ or to $1/\bar{\tau}$ where $\bar{\tau}$ is the mean period of contact between the vapor–condensate packet and the heat-transfer surface as will be shown later.

If we assume that heat is transferred from a packet of the vapor–condensate mixture to the heat-transfer surface by conduction during the contact between them, we have:

$$\frac{\partial T}{\partial \theta} = \alpha_m \frac{\partial^2 T}{\partial y^2} \tag{1}$$

where

$$\alpha_m = \frac{k_m}{\rho_m C_{pm}}$$

y is the distance away from the heat-transfer surface as shown in Fig. 2. It is realistic to consider that the temperature of the packet remains at a constant value of T_b beyond a certain characteristic length from the tube wall, δ, which is the size or thickness of the vapor–condensate packets. The appropriate initial and boundary conditions are

$$\begin{aligned} T &= T_b, \quad \theta = 0, \ y > 0 \\ T &= T_w, \quad \theta > 0, \ y = 0 \\ T &= T_b, \quad \theta > 0, \ y = \delta. \end{aligned} \tag{2}$$

The solution of equation (1) subject to the initial and boundary conditions (2), is available [11] and can be expressed as

$$\frac{T-T_b}{T_w-T_b} = \left[1 - \frac{y}{\delta} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n} \times \left(\sin\frac{n\pi y}{\delta}\right) e^{-\frac{\pi^2 n^2 \alpha_m \theta}{\delta^2}}\right]. \tag{3}$$

Let $q_i(\theta)$ be the instantaneous heat-transfer rate at the wall at a contact time θ. Then,

$$q_i(\theta) = -k_m \left.\frac{\partial T}{\partial y}\right|_{y=0}. \tag{4}$$

The mean heat-transfer rate is the summation of the contributions of all vapor–condensate packets of different ages, i.e.

$$q = \int_0^\infty q_i(\theta)\phi(\theta)\,d\theta \tag{5}$$

where $\phi(\theta)$ is the contact time (or age) distribution function of the vapor–condensate packets. By considering that the vapor–condensate packets on the surface are replaced in a completely random manner [12, 13], because of vigorous agitation of the vapor–condensate mixture in the condenser tube [3], we have

$$\phi(\theta) = s\,e^{-s} \tag{6}$$

where s is the average frequency of surface renewal, i.e.

$$s \equiv \frac{1}{\bar{\tau}} = c_1 \frac{2^N}{\bar{t}}. \tag{7}$$

Substitution of equation (3) into equation (4), and substitution of the resultant expression and equation (6) into equation (5), give the rate of heat transfer across a unit area of the heat transfer surface or wall as

$$q = (T_w - T_b)(s k_m \rho_m C_{pm})^{1/2} \coth\left(\frac{s\delta^2}{\alpha_m}\right)^{1/2}. \tag{8}$$

According to the usual definition of the inside heat-transfer coefficient, h_i, we have

$$h_i = \frac{q}{(T_w - T_b)} = (s k_m \rho_m C_{pm})^{1/2} \coth\left(\frac{s\delta^2}{\alpha_m}\right)^{1/2} = \left(\frac{k_m \rho_m C_{pm}}{\bar{\tau}}\right)^{1/2} \coth\left(\frac{\delta^2}{\bar{\tau}\alpha_m}\right)^{1/2}. \tag{9}$$

For sufficiently large δ, $1/\bar{\tau}$, or $1/\alpha_m$, we have

$$\coth\left(\frac{\delta^2}{\bar{\tau}\alpha_m}\right)^{1/2} \simeq 1. \tag{10}$$

Then, equation (9) reduces to

$$h_i = \left(\frac{k_m \rho_m C_{pm}}{\bar{\tau}}\right)^{1/2} = (C_1 2^N)^{1/2}\left(\frac{k_m \rho_m C_{pm}}{\bar{t}}\right)^{1/2}. \tag{11}$$

To apply this basic surface renewal model as expressed by equation (11), we must know the mean thermal diffusivity, α_m, and the mean residence time of the packets of the vapor–condensate mixture in the tube. The mean residence time, $\bar{t}$, in equation (11) is proportional to the mean velocity of the vapor–condensate mixture, u_m, and to the length of the tube, L. Thus,

$$\bar{t} = C_2 \frac{L}{u_m}. \tag{12}$$

The mean velocity of the vapor–condensate mixture, u_m, can be derived as follows:

For a circular tube with inside diameter, d, and length, L, the volumetric flow rate of the condensate, $\dot{V}_l$, can be written as

$$\dot{V}_l = \bar{u}_l A_l \tag{13}$$

where $\bar{u}_l$ is the mean velocity of the condensate, and A_l, the equivalent cross-sectional area of the condensate flow. Since the vapor and condensate are well mixed and agitated vigorously at any position along the tube, $\bar{u}_l$ and A_l can be written, in general, as

$$\bar{u}_l \simeq u_m \tag{14}$$

$$A_l \propto \delta_{\text{ave}} d \quad \text{or} \quad A_l = C_3 \delta_{\text{ave}} d \tag{15}$$

where δ_{ave} is the average size or thickness of the packets of the vapor–condensate mixture. Note that d and δ_{ave} are two characteristic dimensions of the system, and that δ_{ave} is assumed to be essentially independent of operating conditions. Substituting equations (14) and (15) into equation (13), and solving the resultant expression for the mean velocity of the vapor–condensate mixture, u_m, we obtain

$$u_m = \frac{\dot{V}_l}{C_3 \delta_{\text{ave}} d}. \tag{16}$$

Since the heat removed by the condensation of the vapor is equal to the sum of the latent and sensible heats, we have from the heat balance at any position along the tube

$$Q = -k_m A_r \left.\frac{\partial T}{\partial y}\right|_{y=0} = -[(1-x)H_{fg} + H_{sp}]\dot{M} \tag{17}$$

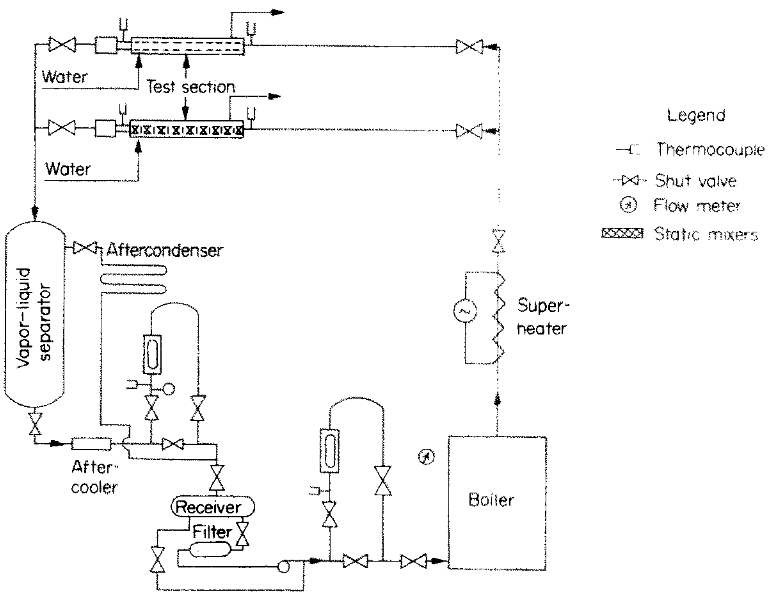

FIG. 3. A schematic diagram for the Freon-113 flow circuits.

where A_r is the overall area of the heat-transfer wall, H_{fg}, the latent heat of condensation, H_{sp}, the sensible heat of superheated vapor, and $\dot{M}$, the mass flow rate of the condensing fluid. We may write

$$-k_m A_r \frac{\partial T}{\partial y}\bigg|_{y=0} \simeq -k_m A_r \frac{\Delta T}{\delta_{\text{ave}}} \simeq -[(1-x)H_{fg}+H_{sp}]\dot{M} \tag{18}$$

where

$$\Delta T = T_b - T_w.$$

Solving equation (18) for δ_{ave}, we obtain

$$\delta_{\text{ave}} = \frac{k_m A_r \Delta T}{[(1-x)H_{fg}+H_{sp}]\dot{M}}. \tag{19}$$

Substitution of equation (19) into equation (16) gives

$$u_m = \frac{\dot{V}_l[(1-x)H_{fg}+H_{sp}]\dot{M}}{C_3\, d k_m A_r \Delta T} \tag{20}$$

where

$$A_r = \pi L d$$
$$\bar{\rho}_l \dot{V}_l = C_4 \dot{M}.$$

Substitution of equations (12) and (20) into equation (11) yields

$$h_i = C\left[\frac{\rho_m C_{pm} \dot{M}^2[(1-x)H_{fg}+H_{sp}]}{d^2 L^2 \bar{\rho}_l \Delta T}\right]^{1/2} \tag{21}$$

where

$$C = \left[\frac{2^N C_1 C_4}{C_2 C_3 \pi}\right]^{1/2}. \tag{22}$$

In dimensionless form, equation (21) becomes

$$\left[\frac{h_i d}{k_m}\right] = C\left[\frac{\mu_m^2 C_{pm}^2}{k_m^2}\,\frac{\dot{M}^2}{\bar{\mu}_l^2 d^2}\,\frac{\bar{\mu}_l^2}{\mu_m^2}\,\frac{d^2}{L^2} \times \frac{\rho m}{\bar{\rho}_l}\,\frac{(1-x)H_{fg}+H_{sp}}{C_{pm}\Delta T}\right]^{1/2},$$

or

$$[Nu] = C[Pr][Re_l]\left[\frac{\bar{\mu}_l}{\mu_m}\right]\left[\frac{d}{L}\right]\left[\frac{\rho_m}{\bar{\rho}_l}\right]^{1/2} \times \left[\frac{(1-x)H_{fg}+H_{sp}}{C_{pm}\Delta T}\right]^{1/2}. \tag{23}$$

COMPARISON OF THE MODEL WITH EXPERIMENTAL DATA AND DISCUSSION

Thirty-two experimental data previously obtained [3] were used here to verify the present model. Figure 3 shows a schematic diagram of the test facility used. It included an electrically heated vapor generator, superheater, vapor–liquid separator, after-condenser, liquid receiver, and a circulating pump. The facility also included two identical test condensers, one of which had 44 static mixers. They were mounted horizontally in parallel. The inside diameter of each condenser tube was 1.27 cm and its length was 61 cm. Each condenser was a counter-current double pipe heat exchanger inside which refrigerant-113 condensed in the tube while the cooling water flowed in the annulus. The test condensers were instrumented to measure the inlet and outlet temperatures and pressures of the condensing fluid, and the inlet and outlet temperatures of the coolant and its flow rate. All temperatures were measured by copper–constantan thermocouples. The pressure drop across the test section was measured by a Foxboro differential pressure cell Type 13A. The average inside condensation heat-transfer coefficient h_i for the tubes with and without the static mixer was calculated as follows:

The overall heat-transfer rate equation for the experimental test condenser is given by

$$Q = \frac{A_i \Delta T}{\dfrac{1}{h_i} + \dfrac{A_i \ln(D/d)}{2\pi K L} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}}. \tag{24}$$

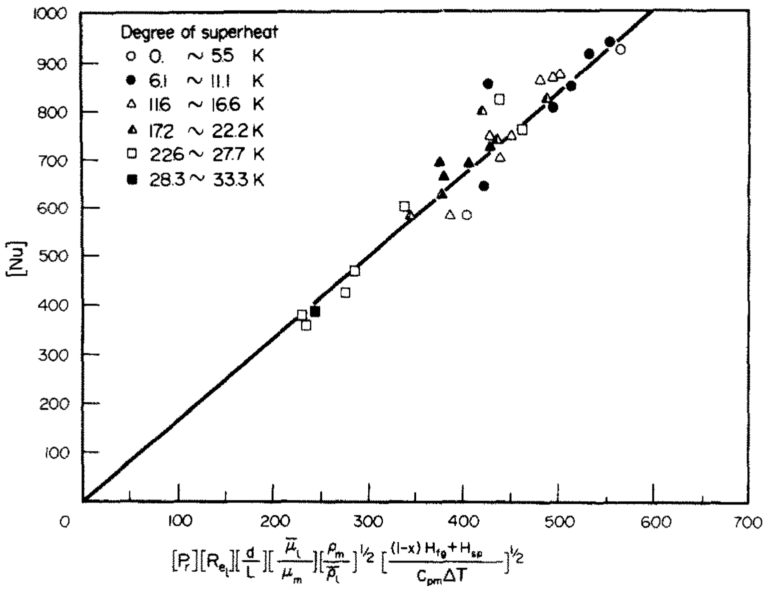

FIG. 4. Correlation of experimental data [3] with equation (23) of the present model.

The heat-transfer rate Q was calculated from the mass flow rate of the cooling water and its temperature rise. The ranges of operating conditions over which the data were obtained were:

1. Condensation temperature, 327.4–349.6 K.
2. Inlet superheats, 5.5–29.3 K.
3. Condensing pressure, 1.17×10^5–2.62×10^5 N/m^2.
4. Condensing fluid mass velocity, 33.3–102.7 kg/m^2 s.

Figure 4 plots, according to equation (23), the experimental data of condensation inside the tubes with static in-line mixer inserts.

A linear regression analysis of the data based on equation (23) yielded a value $C = 1.78$ with a correlation coefficient, r, of 0.96, and a standard deviation, $S_{y \cdot x}$, of 25.5. A test of linearity of the data by an analysis of variance involving the F-test [14], and a test of the null hypothesis, that the regression line passes through the origin by the t-test [15], indicate that such a linear relation adequately correlates the data. The slope C of the regression line is a function of the number of static mixers, exposure time and other factors, as shown in equation (23).

CONCLUSION

A surface renewal model has been proposed to describe the condensation heat transfer inside a circular tube with in-line static mixers. A dimensionless correlation has been developed based on the model. It includes a correlation constant which has been determined from experimental measurements. Additional data are needed to verify the applicability of the correlation to other fluids.

Acknowledgement—This work was supported by the Engineering Experiment Station, Kansas State University.

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MODELE DE RENOUVELLEMENT DE SURFACE POUR LE TRANSFERT THERMIQUE PAR CONDENSATION DANS DES TUBES DANS DES MELANGEURS STATIQUES EN LIGNE

Résumé—Un modèle de renouvellement de surface est développé pour le transfert thermique par condensation dans un tube circulaire avec des mélangeurs statiques en ligne. Ce modèle conduit à la formulation adimensionnelle suivante:

$$[Nu] = c[Pr][Re_l]\left[\frac{\mu_l}{\mu_m}\right]\left[\frac{d}{L}\right]\left[\frac{\rho_m}{\rho_l}\right]^{1/2}\left[\frac{(1-x)H_{fg}+H_{sp}}{C_{pm}\Delta T}\right]^{1/2}$$

La constante C est déterminée expérimentalement.

EIN OBERFLÄCHENBILDUNGS-MODELL FÜR DIE WÄRMEÜBERTRAGUNG DURCH KONDENSATION IN ROHREN MIT FESTSTEHENDEN, HINTEREINANDER ANGEORDNETEN EINBAUTEN

Zusammenfassung—Ein Modell wurde entwickelt, welches die Bildung der Oberfläche beim Wärmetransport durch Kondensation in Kreisrohren mit feststehenden, in Reihe angeordneten Einbauten beschreibt. Dieses Modell liefert die folgende dimensionslose korrelierte Gleichung:

$$[Nu] = C[Pr][Re_l]\left[\frac{\bar{\mu}_l}{\mu_m}\right]\left[\frac{d}{L}\right]\left[\frac{\rho_m}{\bar{\rho}_l}\right]^{1/2}\left[\frac{(1-x)H_{fg}+H_{sp}}{C_{pm}\Delta T}\right]^{1/2}$$

Die Konstante C wurde experimentall bestimmt.

МОДЕЛЬ ВОССТАНОВЛЕНИЯ ПОВЕРХНОСТИ КОНДЕНСАЦИОННОГО ТЕПЛООБМЕНА В ТРУБАХ С УСТАНОВЛЕННЫМИ СООСНО СТАТИЧЕСКИМИ СМЕСИТЕЛЯМИ

Аннотация — Модель восстановления поверхности разработана для конденсационного теплообмена в курглой трубе с установленными соосно статическими смесителями. Эта модель позволила вывести следующее безразмерное корреляционное уравнение:

$$[Nu] = C[Pr][Re_l]\left[\frac{\bar{\mu}_l}{\mu_m}\right]\left[\frac{d}{L}\right]\left[\frac{\rho_m}{\bar{\rho}_l}\right]^{1/2}\left[\frac{(1-x)H_{fg}+H_{sp}}{C_{pm}\Delta T}\right]^{1/2}$$

Константа C определена экспериментально.